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# Theoretical analysis of extensive air showers IV. Muon showers deep underground

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**Abstract.** Studies of muon showers deep underground by the Utah group have stimulated an extension of the previous calculations of the present authors into the region covered by the underground experiment: viz. muon threshold energies of the order of 1000 GeV and zenith angles in the region of 60°.

The effect of varying some of the more important parameters of the high-energy nucleon–air-nucleus collision has been examined, and it is concluded that a multiplicity law of the form $E_p^{1/4}$ is preferred to an $E_p^{1/2}$ law and that there is no evidence for a significantly greater mean transverse momentum than the value of 0·4 GeV/*c*, which relates to the parents of muons of lower energy. An increase in the fraction of kaons generated with increasing nucleon energy is not favoured.

Concerning the mass spectrum of the primary particles, there is some support for the contention that the primaries above $10^{15}$ eV are largely protons, but none for the conclusion of Grigorov *et al.* in 1967 that the proportion of heavy nuclei commences to increase at $10^{12}$ eV. However, a problem that affects all the conclusions is that significantly fewer multiple events are observed than expected.

## 1. Introduction

This paper continues a series on the theoretical aspects of extensive air shower phenomena using a conservative model for the various nuclear interaction processes in the atmosphere. The earlier results have been published by de Beer *et al.* (1966, 1968, 1969, to be referred to as I, II and III, respectively).

The last mentioned paper, III, concerned the theory of extensive air shower phenomena at large angles to the vertical, where the electron component is almost completely absorbed and only the muons remain. The results were used in a later paper, by Rogers *et al.* (1969), to analyse the data from an experiment at ground level in which the density spectrum of multiple muons had been measured. It was shown that the model was reasonable for primary nucleon energies below about $10^{15}$ eV, where the mass composition of the primaries is thought to be known.

At ground level the mean muon energy in the detected showers in the work of Rogers *et al.* is of the order of tens of GeV and the majority come from late interactions in the nuclear-active cascade, where the energy of the particle (pion) producing the interaction from which the detected muons' parents are derived is quite modest. Very recently, however, data have been reported by Porter and Stenerson (1969) which refer to muons of much higher energy and thus to interactions of greater energy, and it is clearly desirable to examine the applicability of our conservative model under these new conditions.

The general problem of the parameters of interest, which can be examined from studies of high-energy muon showers as a function of zenith angle, are examined in a later paper; in the present work attention will be confined to calculations of relevance to the results given by Porter and Stenerson. In particular, the problem of the form of the relation between the number of pions produced and the energy of the interacting particle will be examined.

## 2. Experimental results on muon showers underground

### 2.1. *The experimental arrangement at Utah*

The Utah apparatus, which was essentially designed to detect neutrino-induced muons, is located in an underground chamber 1850 ft below the surface of a mountain.

The completed detector, consisting of four directional Čerenkov counters, an array of 600 cylindrical spark counters and two 16 kG solid iron magnets, has dimensions 12 m × 11 m × 6 m in height.

In the initial experiments in 1967 only slightly more than half the detector was operational but, nevertheless, a large amount of data accrued on the atmospheric muon component. Considerable interest has been aroused by the observations of the angular distribution of single muons (see, for example, Bergeson *et al.* 1968), but attention is directed here to the results on groups of multiple muons, that is the residue of extensive air showers.

The data to be used are summarized in table 1, where approximate ground-level muon threshold energies calculated by the present authors are also given. A useful feature of the results is that there are data for a range of muon threshold energies in each band of zenith angle, a situation which arises from the irregular terrain above the detector.

**Table 1. Distribution of multiple muons in Utah experiment (after Porter and Stenerson 1969)**

| Zenith angle (deg) | Depth range (m.w.e.) | Approx. $E_{\mu t}$ (GeV) (threshold) | Multiplicity 1 | 2 | 3 | 4 | >4 |
|---|---|---|---|---|---|---|---|
| 40–50 | 1400–1900 | 540 | 18 | 27 | 3 | 4 | 0 |
| | 1900–2400 | 800 | 1831 | 153 | 40 | 5 | 9 |
| | 2400–2900 | 1100 | 2465 | 157 | 35 | 10 | 10 |
| | 2900–3400 | 1500 | 281 | 14 | 0 | 1 | 1 |
| 50–60 | 1900–2400 | 800 | 48 | 14 | 0 | 0 | 0 |
| | 2400–2900 | 1100 | 2315 | 107 | 17 | 4 | 6 |
| | 2900–3400 | 1500 | 1756 | 86 | 12 | 3 | 0 |
| | 3400–3900 | 1900 | 629 | 31 | 2 | 3 | 0 |
| | 3900–4400 | 2400 | 39 | 2 | 1 | 0 | 0 |
| 60–70 | 2400–2900 | 1100 | 19 | 1 | 0 | 0 | 0 |
| | 2900–3400 | 1500 | 834 | 25 | 3 | 1 | 1 |
| | 3400–3900 | 1900 | 1074 | 32 | 9 | 0 | 0 |
| | 3900–4400 | 2400 | 471 | 18 | 2 | 0 | 0 |
| | 4400–4900 | 3000 | 215 | 11 | 0 | 1 | 0 |
| | 4900–5400 | 3700 | 35 | 4 | 0 | 0 | 0 |
| | 5400–5900 | 4600 | 21 | 0 | 0 | 0 | 0 |
| 70–75 | 3900–4400 | 2400 | 99 | 1 | 0 | 1 | 0 |
| | 4400–4900 | 3000 | 138 | 4 | 1 | 0 | 0 |
| | 4900–5400 | 3700 | 107 | 3 | 0 | 0 | 0 |
| | 5400–5900 | 4600 | 25 | 1 | 0 | 0 | 0 |
| | 5900–6400 | 5700 | 9 | 0 | 0 | 0 | 0 |
| | 6400–6900 | 6800 | 11 | 1 | 0 | 0 | 0 |

### 2.2. *The empirical muon density spectra*

In the absence of theoretical predictions of the expected frequencies of multiple muons Porter and Stenerson determined empirical integral density spectra, which, when folded in with the various geometrical acceptance functions, gave a best fit to the observed results. These empirical spectra are indicated in figure 1, where the median densities corresponding to the various multiplicities are shown. An indication is also given of the densities and threshold energies below which the statistical accuracy of the data is good.

In what follows the density spectra expected for our conservative theory with alternative forms of the multiplicity law are calculated, allowance being made for the non-negligible detecting area of the Utah detector, and the resulting spectra are compared with the results of figure 1.

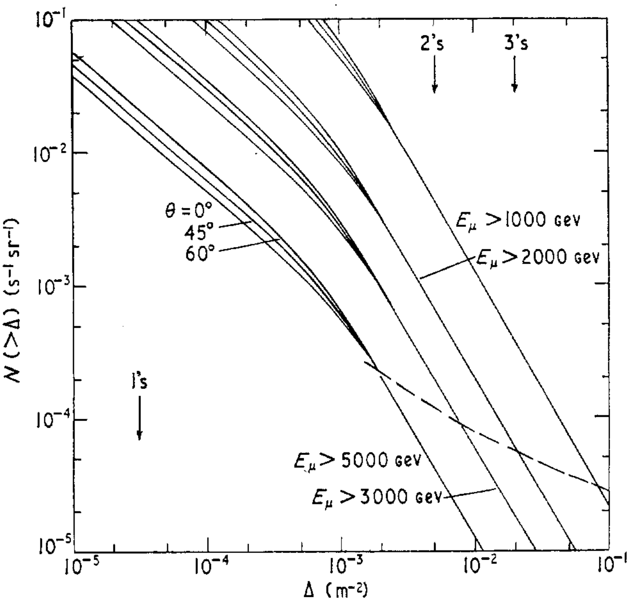


Figure 1. Empirical density spectra from the Utah experiment (after Porter and Stenerson 1969).

## 3. Predictions of the model

### 3.1. *The essential details of the model and the form of the calculations*

The full details of the model can be found in I. Of the parameters likely to have most effect on the predicted muon density spectra the multiplicity law (that is, the relationship between number of pions produced and the energy $E_p$ of the initiating pion or nucleon) and the mean value of the transverse momentum ($\langle p_t \rangle$) of the secondary pions are probably the most important. The situation regarding multiplicity is as follows.

At primary energies below about 3000 GeV direct experimental data suggest an $E_p^{1/4}$ law, but at higher energies the data are so sparse and uncertain that widely differing variations have been suggested. For example, Orford and Turver (1968) have shown that their results on the lateral distribution of energetic muons ($10 < E_\mu < 100$ GeV) in showers of mean size of approximately $10^7$ particles can be understood in terms of an $E_p^{1/2}$ law above 3000 GeV. At the other extreme Pinkau (1966) has given arguments which favour a ln $E_p$ variation. In the present work calculations are given for our preferred conservative model (the model used in I, II and III), that the multiplicity varies as $E_p^{1/4}$ for all interaction energies and, as an alternative, for the $E_p^{1/2}$ law. If we put in the numerical coefficients the alternative multiplicity laws used in the calculations are as follows:

(i) $n_s = 2{\cdot}7E_p^{1/4}$ with $E_p$ in GeV—the '$E^{1/4}$ law'.

(ii) $n_s = 2{\cdot}7E_p^{1/4}$ to $E_p = 3000$ GeV, followed by $0{\cdot}36E_p^{1/2}$ at higher energies—the '$E^{1/2}$ law'.

If we turn to the problem of mean transverse momentum, accurate calculations have been made for a value of 0·4 GeV/$c$. What direct evidence there is suggests a slowly varying mean, increasing from about 0·35 GeV/$c$ at $10^{10}$ eV to 0·5 GeV/$c$ at $10^{14}$ eV (de Beer *et al.* 1968), and allowance has been made for this variation by a relaxation method.

The displacement of muons by Coulomb scattering in the earth can be shown to be negligible compared with that due to the transverse momentum of the parent pions and it is disregarded here. Similarly, if attention is confined to angles below about 65° the effect of geomagnetic deflection can be disregarded.

The calculations were carried out in three stages:

(i) The average numbers of muons expected at sea level for various threshold energies were calculated for a variety of primary energies, averaging appropriately over interaction points in the atmosphere. The calculations were made for a zenith angle $\theta = 60°$, this being roughly the median angle in the Utah experiment, and for the two multiplicity laws. The primary spectrum derived in III was then used to determine the energy spectrum of single muons, and this spectrum was compared with the Utah results and with other measurements.

(ii) Muon lateral distributions were determined, applicable both to point detectors and to the Utah array, which had, for the measurements reported by Porter and Stenerson, an average area normal to the muon beam at 60° of 20 m$^2$.

(iii) The lateral distributions were combined with the primary spectrum to give the predicted integral density spectra for a variety of muon threshold energies, and these were then compared with the empirical spectra derived by Porter and Stenerson. The comparisons were made for both multiplicity laws and for alternative assumptions about the primary mass composition above $10^{15}$ ev. Small corrections were applied to account for the contribution from kaons and the slow variation of $\langle p_t \rangle$ with interaction energy. Detailed calculations were not made for a variety of zenith angles because the variation of density spectrum with angle is almost entirely a simple geometrical consequence of the increased path length to the interaction points, with some effect from changed pion decay probability.

### 3.2. *The predicted energy spectrum of single muons*

The results on the mean number of muons expected at a zenith angle of 60° as a function of primary nucleon energy are given in figure 2 for the two multiplicity laws. When combining with the primary nucleon spectrum there is the usual problem of which

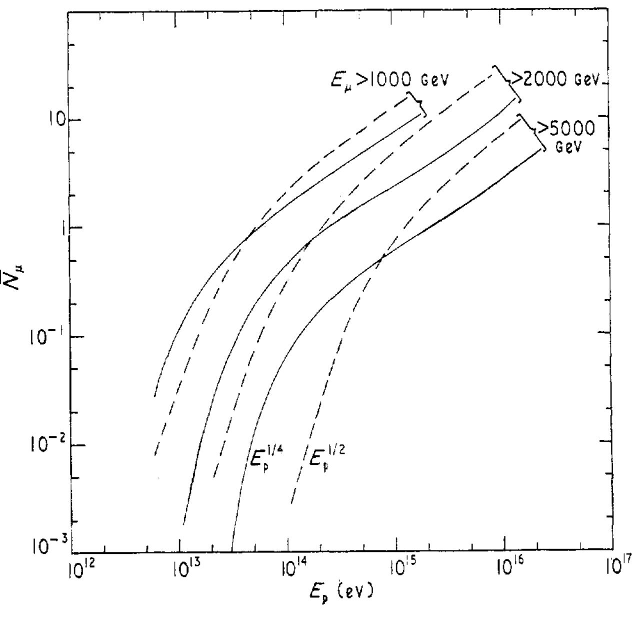


Figure 2. Mean muon number $\bar{N}_\mu$ against primary nucleon energy $E_p$ for $\theta = 60°$ and pions only.

spectrum to adopt, since it has not been measured directly in the relevant energy region. In the present work, with the $E^{1/4}$ multiplicity law, the spectrum derived in III is used (extrapolated back to lower energies), this procedure having some justification since it was derived from extensive air shower data—the measured sea-level shower size spectrum —using our model. When we express the energy in GeV/nucleon, the spectrum is given by $j(E) = 8{\cdot}4 \times 10^3 E^{-2{\cdot}6}$ m$^{-2}$ s$^{-1}$ sr$^{-1}$ GeV$^{-1}$ (or $2{\cdot}1 \times 10^{18} E^{-2{\cdot}6}$ m$^{-2}$ s$^{-1}$ sr$^{-1}$ ev$^{-1}$ for

$E$ in ev), for $E < 10^{15}$ ev. At higher energies there is the problem of the mass composition of the primaries and alternative spectra have been taken: the 'modulated' spectrum in which galactic modulation causes the average mass to increase, and the 'protons-only' spectrum, in which only protons remain (see III for details).

With the $E^{1/2}$ law, the primary intensity at $10^{15}$ ev and above must be increased to preserve agreement with the measured size spectrum (see I); the spectrum is raised by a factor 1·6 at $10^{15}$ ev but made to pass through the same point at $10^{10}$ ev, since it has been measured directly in this region.

It is necessary to note that the adopted intensities in the region of $10^{10}$ ev are below those directly measured by a factor of about 2. They are similarly below the energy per nucleon spectrum derived by Brooke *et al.* (1964), which was based on sea-level measurements of muon and proton spectra and used a similar interaction model to that adopted here. This apparent inconsistency is due to the fact that in the work on Brooke *et al.* allowance was made for interaction energy which was 'lost' and which did not contribute to the pion–muon component. There the difference between the total and pion inelasticities was taken to be 0·12 (a 'wastage' of 0·12/0·35, i.e. 34%). In III, on the other hand, the loss of energy was neglected. In fact, some of the energy in question will be given to kaons and, whereas in III and in the work of Brooke *et al.* (at primary energies below $10^{12}$ ev) these kaons did not contribute greatly to the low-energy muon flux considered, in the present case they will have a significant effect. Allowance is made here by adding in a contribution from kaons corresponding to a $K/\pi$ ratio of 20% (Osborne and Wolfendale 1964) over and above the full pion component; this is equivalent to taking a somewhat higher primary spectrum.

The resulting single-muon spectra are shown in figure 3 for the two multiplicity laws with pions only and for the $E^{1/4}$ law with an additional component from kaons.

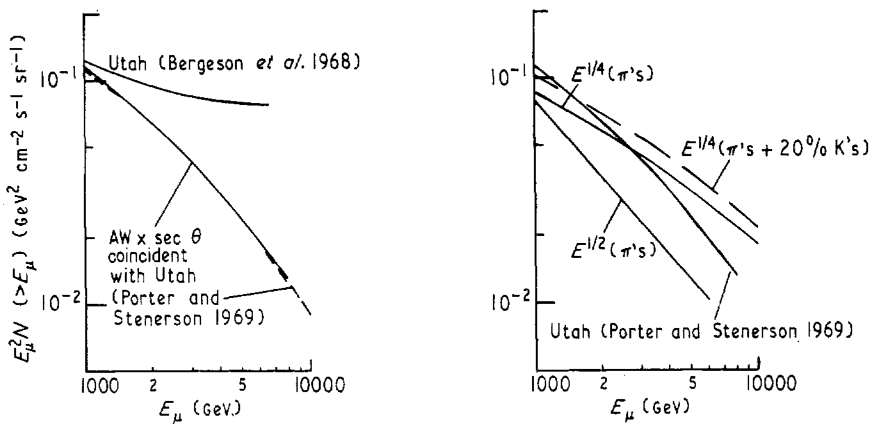


Figure 3. Single muon spectra for $\theta = 60°$ (AW, Aurela and Wolfendale 1967).

A problem arises when a comparison is made with the Utah results because two muon spectra have been quoted for this experiment. The first, due to Bergeson *et al.* (1968), is derived from the single-muon measurements—data which have caused such interest because of the suggestion of a new source of muons. These authors argue that a consequence of the measurements is that the rate of energy loss of energetic muons is greater than had hitherto been thought, and the result is that when the depth–intensity variation is taken as the datum the derived sea-level muon spectrum falls off less rapidly with energy, and has higher intensities than had previously been accepted. Even when allowance is made for the slower increase of intensity with zenith angle than usually accepted, the resulting 60° spectrum is still of higher intensity than the 'conventional' spectrum (typified by the spectrum shown and attributed to Aurela and Wolfendale (1967).

The second Utah muon spectrum comes from the work of Porter and Stenerson, who use the Utah single-particle data, together with the *conventional* rateof energy loss, to determine the shape of the integral density spectrum at small densities. Integrating the differential spectrum $F_\theta\,(\Delta,\, E_\mu > E_{\mu t})$ appropriately, we find the integral muon spectrum:

$$N_\theta\,(>E_{\mu t}) = \int_0^\infty F_\theta(\Delta,\, E > E_{\mu t})\Delta\, d\Delta.$$

Rather fortuitously, perhaps, for $\theta = 60°$ this spectrum agrees almost exactly with that derived from the vertical spectrum of Aurela and Wolfendale (1967) by multiplying by sec $\theta$, a procedure that assumes that the majority of muons are derived from pions and that none comes from any new process.

In so far as we use the data on multiple muons given by Porter and Stenerson, we shall also adopt their single-muon spectrum.

Comparing now with the predictions of our model, we see that for absolute intensities the second Utah spectrum is rather close to the prediction for $E^{1/4}$ and pions alone, but for shape the $E^{1/2}$ law is superior. If kaons are added, the $E^{1/2}$ law is in better agreement both with regard to shape and intensity. At this stage, however, the evidence is insufficient to choose between the multiplicity laws.

### 3.3. *Muon lateral distribution*

The calculated lateral distributions for a threshold muon energy $E_{\mu t}$ of 1000 GeV are shown in figure 4, the majority being for the $E^{1/4}$ law. Distributions for the $E^{1/2}$ law for two primary energies are also given and it is seen that, as expected, the behaviour of

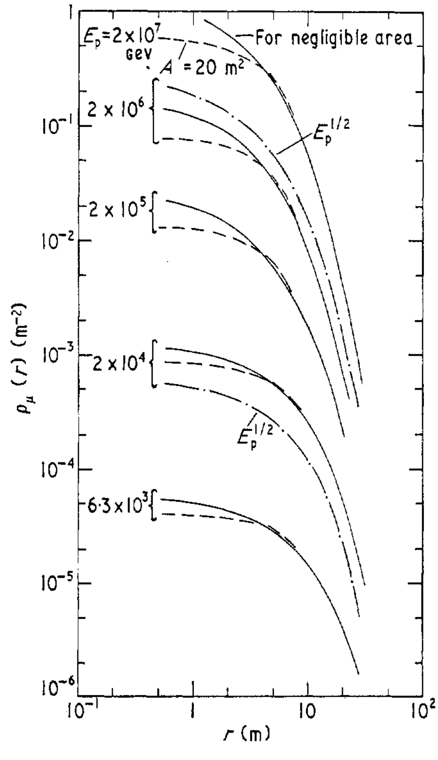


Figure 4. Muon lateral distributions for $E_\mu > 1000$ GeV. $\theta = 60°$, $\pi$ only, $\langle p_t \rangle = 0{\cdot}4$ GeV/$c$, $E_p^{1/4}$ law unless stated otherwise.

figure 2 is reflected, that is, the $E^{1/2}$ intensities are higher at higher primary energies and lower at the lower values of $E_{\mu t}$. At the higher energy, furthermore, the more rapid degradation of the primary energy with its attendant high muon origins causes a slight widening of the distribution. Over the important range of radial distance $2 < r < 15$ m the lateral distributions can be represented to a first approximation by a relation of the form $\rho_\mu(r) = A \exp(-r/r_0)$, where $r_0$ is a slowly varying function of $E_p$ and a more rapidly varying function of zenith angle and muon threshold energy. Confining attention to showers which give two detected particles, for $1000 < E_{\mu t} < 2000$ GeV we find that the relationship is

$$r_0 \propto \sec^{1\cdot 3}\theta, \text{ with } 45° < \theta < 60°$$

and for the same angular range

$$r_0 \propto E_{\mu t}^{-0\cdot 8}, \text{ with } 700 < E_{\mu t} < 3000 \text{ GeV}.$$

In both cases primary protons are assumed, together with $\langle p_t \rangle = 0\cdot 4$ GeV/$c$ and the $E_p^{1/4}$ multiplicity law.

The calculations referred to so far have related to the muon density at a point; when applied to the case of a detector having an area comparable with the average displacement of the muons from the shower axis a correction is necessary. Calculations have been made for the effective area of the Utah array (2 m × 10 m (Stenerson, private communication)) by considering shower axes distributed at various distances from the centre of the array and by finding the appropriate mean density over the array. The resulting lateral distributions are shown as broken lines in figure 4; understandably there are significant differences for radial distances less than the average dimension of the array.

### 3.4. *The integral density spectra*

The lateral distributions have been folded in with the primary spectrum of III (for the case of a modulated composition, i.e. an increased flux of heavy nuclei above $10^{15}$ ev) to give the predicted integral spectra for the various threshold energies. Results for the 'basic' parameters, $\langle p_t \rangle = 0\cdot 4$ GeV/$c$ and pions only, are shown in figure 5, where comparison is made with the Utah empirical spectra.

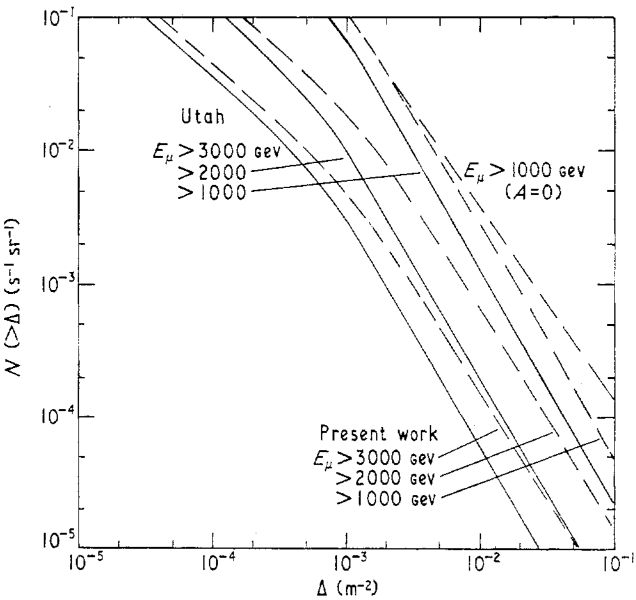


Figure 5. Integral density spectra—comparison with Utah empirical spectra. Modulated spectrum, $\pi$ only, $\langle p_t \rangle = 0\cdot 4$ GeV/$c$, $E_p^{1/4}$ law.

The density spectra are indicated both for the case of a point detector and the Utah detector with its normal area of 20 m$^2$; the effect of the finite area is seen to be considerable at high densities.

A more detailed comparison is given in figure 6 of the density spectra for the important muon threshold energies, to which most of the experimental data refer. The comparison

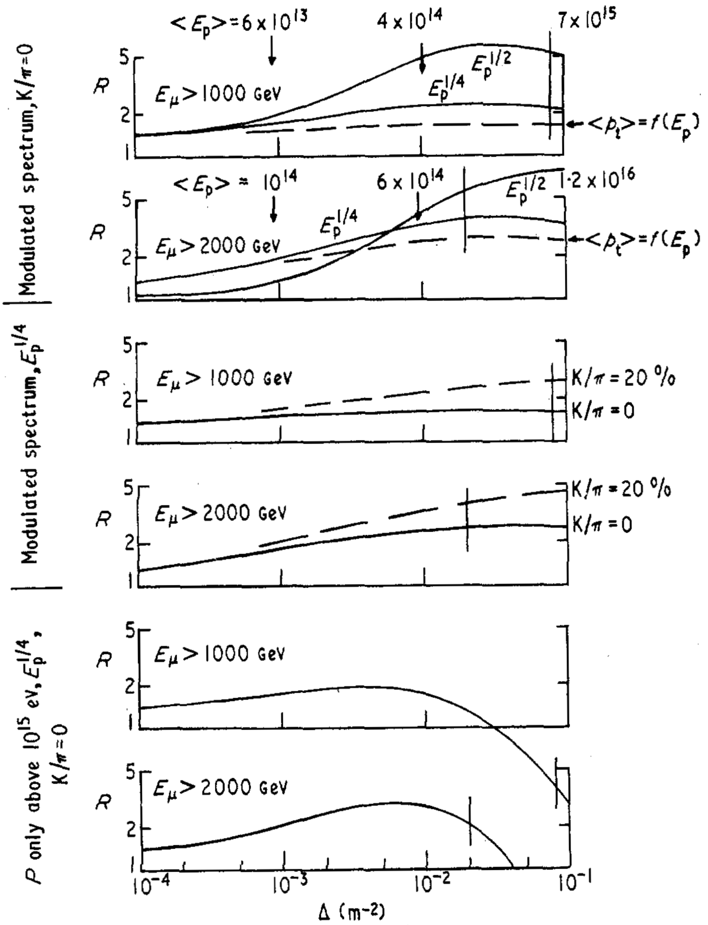

Figure 6. Ratio $R$ of calculated integral density spectrum to Utah empirical spectrum. (The short vertical lines mark the end of the region of good statistical accuracy.)

is made for the various combinations of parameters for which calculations have been made. On the families of curves for the modulated primary spectrum the median primary energies are indicated for the $E^{1/4}$ multiplicity law.

Calculations have been made of the effect of kaons, on the assumption that $\langle p_t \rangle$ is the same as that for pions. There is evidence (Bigi *et al.* 1962), however, that this is not the case and that the value for kaons is greater than that for pions; thus the contributions indicated are upper limits to the intensity.

## 4. Discussions and conclusions

Examination of figure 6 shows that for most of the density range considered the predicted intensities are higher than those observed experimentally. If we consider first the situation where the modulated spectrum is used, it is clear that the fit with experiment is inferior for the $E^{1/2}$ law. With the $E^{1/4}$ law the discrepancy is smaller but still serious, even when the reduction which comes from the slow increase in $\langle p_t \rangle$ with primary energy is allowed for. If kaons are included, the discrepancy is even more marked.

If the primary particles are assumed to be only protons above $10^{15}$ ev, the predicted intensity falls rapidly at densities above $10^{-2}$ m$^{-2}$, so rapidly as to predict a lower intensity than observed, although the range of observed densities for which this occurs is small and this is only true if kaons are neglected.

In order to achieve better agreement between observation and expectation a number of possibilities can be considered.

### 4.1. *Transverse momentum*

If the value of $\langle p_t \rangle$ were to increase with increasing interaction energy the predicted intensities would be lowered and the reduction factor would become larger as the density under consideration was raised and as the muon threshold energy increased. Near agreement with observation would be achieved if $\langle p_t \rangle$ were increased to about 1 GeV/*c* at an interaction energy of about $10^{15}$ eV, a result which rather fortuitously is similar to the suggestion of de Beer *et al.* (1968) as a means of explaining the unusually wide lateral distribution of energetic muons found by Earnshaw *et al.* (1967).

However, a contradiction would arise with data from the Utah experiment, not hitherto mentioned, which concerns the average separation of the detected muons. Porter and Stenerson quote an r.m.s. radius for muons of threshold energy 1000 GeV in the region of 7 m for a zenith angle of 50°, and this can be compared with a predicted value of about 10 m from the present calculations with $\langle p_t \rangle = 0{\cdot}4$ GeV/*c*. If $\langle p_t \rangle$ were to increase, the corresponding r.m.s. radius would increase in proportion and a significant divergence from observation would result. Thus, although an increase in $\langle p_t \rangle$ cannot be ruled out, it seems unlikely.

### 4.2. *'Direct' production*

If direct production does occur, to the extent suggested by Bergeson *et al.* (1968), the muon energy scale in the comparison will be affected. The alteration is in the direction to depress the ratios in figure 6 somewhat, the reduction becoming more marked the higher the threshold muon energy and the higher the density. If we turn to the single-muon spectrum (figure 3), the extra muons produced 'directly' would add to those produced by the pions so as to give a result nearer the observed spectrum of Bergeson *et al.*

However, the intensities at the higher values of $\Delta$ ($\Delta \gtrsim 10^{-3}\,\mathrm{m}^{-2}$) are still too high and the results cannot be regarded as providing evidence for (or against) the 'direct' production hypothesis.

### 4.3. *Varying primary composition*

If the transition from a mixed primary composition to a pure proton beam were made more gradual, it would be possible to achieve better agreement with experiment, at least for the $E^{1/4}$ law.

### 4.4. *Reduction of inelasticity of interactions*

A possible way of reducing the muon densities by a large factor is to reduce the inelasticity of the interactions, and thereby to increase the atmospheric density in the region where the important interactions occur—thus more pions would interact and fewer decay into muons. However, there is no evidence from other experiments for such a change in inelasticity and, indeed, difficulties would almost certainly arise in explaining observed shower absorption characteristics.

In conclusion, concerning the multiplicity law, what evidence there is from this preliminary analysis favours an $E^{1/4}$ law rather than the $E^{1/2}$ variant. So far as the mass composition is concerned, there is some evidence in favour of a trend towards a protonic mass composition above $10^{15}$ eV, but better agreement results if the phasing out of heavy primaries takes place over a more extended energy region than has been considered so far. At lower energies the results do not support the conclusion of Grigorov *et al.* (1967), from observations with the 'PROTON' satellites, that the proportion of heavy nuclei in the primary beam commences to increase at $10^{12}$ eV.

Finally, there is strong objection to an increasing rate of production of kaons with increasing energy, unless the mean transverse momentum of these particles significantly exceeds that of pions.

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